Skills from previous math classes that you need to self-review for Math 2A

From Algebra:

Negative and fractional exponents
Rational expressions
- Add / subtract
- Polynomial long division

From Precalculus:

Finding zeros & factoring polynomials
Variation / proportionality
Recurrence relations

From Calculus:

Limits involving infinity
Continuity & differentiability (single & multivariable functions)
Differentiation (basic, product / quotient / chain rule)
Interpretation of derivatives (and their relationship to increasing/decreasing behavior of functions)
Integration (basic, substitution, by parts, partial fraction decomposition, improper, multivariable)
Power series

<u>You must be able to solve these</u>
<u>using neither your calculator nor a table of integrals nor any external aid</u>
<u>All answers must be completely simplified</u>

[1] Over what interval(s) are the following functions continuous ? Over what interval(s) are they differentiable ?

[a] $f(x) = x^3$ [b] $f(x) = x^{-2}$ [c] $f(x) = x^{\frac{1}{3}}$ [d] $f(x) = x^{\frac{3}{2}}$

[e] $f(x) = e^{3x}$ [f] $f(x) = 2^{-3x}$ [g] $f(x) = \ln x$

[h] $f(x) = \sin x$ [i] $f(x) = \tan x$ [j] $f(x) = \csc x$

[k] $f(x) = \dfrac{x+1}{(x-2)(x^2+9)}$

[2] [a] If $y = x^2 e^{-3x}$, find $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} - 2y$.

[b] If $y = x^{\frac{4}{3}}$, find $3x^2 y'' - 4xy' + 4y$. For what values of x is your answer correct ?
(HINT: Consider domains of all functions.)

[c] If $y = Ae^{2x}\sin 3x + Be^{2x}\cos 3x$, find $\dfrac{d^3y}{dx^3} - 4\dfrac{dy}{dx} - 9y$.

[3] Factor each polynomial below
[i] into a product of linear and/or irreducible quadratic factors, and also
[ii] into a product of only linear factors

[a] $3x^3 - 10x^2 + 7x + 10$

[b] $6x^5 - 5x^4 - 24x + 20$

[4] Find all zeros (real and complex) of the polynomial $x^5(2x^2 - 3x - 1)^2(x^2 - 2x + 3)^3$, and find the multiplicity of each zero.

[5] Evaluate the following integrals.

[a] $\int e^{-2x}\sin 3x\, dx$ [b] $\int x^3\cos 2x\, dx$ [c] $\int \dfrac{1}{1+4x}\, dx$ [d] $\int \dfrac{1}{4+x^2}\, dx$

[e] $\int \dfrac{1}{4-x^2}\, dx$ [f] $\int \dfrac{x}{4-x^2}\, dx$ [g] $\int \dfrac{x^2}{4-x^2}\, dx$ [h] $\int \dfrac{3x^4 - 5x^3}{4-x^2}\, dx$

[i] $\int \dfrac{100}{(4+x)^2(4+x^2)}\, dx$ [j] $\int \dfrac{x^3}{y}\, dy$ [k] $\int \dfrac{x^3}{y}\, dx$

[6] [a] If $-\ln y = 2\ln x + C$, and $y = 4$ when $x = 3$, write y as a completely simplified function of x.

[b] If $y(x) = Ae^{2x}\sin 3x + Be^{2x}\cos 3x$, and $y(\frac{\pi}{2}) = -4$ and $y'(\frac{\pi}{2}) = 1$, find the values of A and B.

[c] If $\dfrac{dy}{dx} = \dfrac{1}{\sqrt[3]{x}}$ and $y(8) = -1$, write y as a completely simplified function of x.

[7] If $f(x, y) = \dfrac{3y - 2x}{5x + 4y}$, find f_x and f_y. Find all (x, y) where f_y is continuous.

[8] If $\dfrac{\partial f}{\partial x} = \dfrac{3y - 2x}{5x + 4y}$, find $\dfrac{\partial^2 f}{\partial x \partial y}$ and $\dfrac{\partial^2 f}{\partial y \partial x}$.

[9] If $f'(x) = \dfrac{e^x}{x}$, and $f(2) = -1$, find the slope-point equation of the tangent line to $y = f(x)$ at $x = 2$.

[10] Write equations corresponding to the following situations. **(NOTE: Do NOT solve the resulting equations.)**

[a] The net wealth of a person is proportional to the cube of their age.

[b] The reaction time of a subject varies inversely with the square root of their weight.

[c] The rate of change of a population of rabbits varies directly with the square of the population.

[d] The rate at which you acquire knowledge in a subject varies directly with the difference between your current knowledge and the total knowledge possible (which is a fixed constant).

[11] If $u = \ln y$, use the chain rule to write $\dfrac{dy}{dx}$ in terms of x, u and $\dfrac{du}{dx}$.

(NOTE: Your final answer must NOT involve *y*.)

[12] If $\dfrac{dy}{dx} = \dfrac{4-x^2}{(x+1)^2}$, find all intervals over which y is increasing.

(NOTE: Do NOT solve for *y*.)

[13] Find $\displaystyle\int_0^\infty x^2 e^{-3x}\,dx$.

[14] If $a_{n+2} = na_{n+1} - 3a_n$ for $n \in Z^{non-neg}$, and $a_0 = -1$ and $a_1 = 2$, find a_4.

[15] Find the horizontal asymptote(s) of $f(x) = \dfrac{7}{3+2e^{-5x}}$.

[16] [a] Write the first five non-zero terms of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{7\cdot 4\cdot 1\cdots(10-3n)}{(n-1)!}x^{2n}$.

[b] Write the power series $\dfrac{2}{1\cdot 2}x - \dfrac{4}{1\cdot 2\cdot 3}x^3 + \dfrac{8}{1\cdot 2\cdot 3\cdot 4}x^5 - \dfrac{16}{1\cdot 2\cdot 3\cdot 4\cdot 5}x^7 + \dfrac{32}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6}x^9 - \cdots$ in sigma notation using 0 as the lower limit of summation.

HINT:
The questions above involve material from the following classes.

Precalculus I:	Math 31	[3], [4], [6a], [10]
Calculus I:	Math 1A	[1], [2], [6b], [9], [11], [12], [15]
Calculus II:	Math 1B	[5], [6c], [13]
Calculus III:	Math 1C	[14], [16]
Calculus IV:	Math 1D	[5], [7], [8]

NOTE:
There is no solution key for this prerequisite package since it only involves material that you have learned before.
You are encouraged to work together with your classmates, and to consult your old textbooks and notes.
Feel free to ask me to look over your solutions, or to direct you to relevant sections in your old textbooks.
However, I will not give solutions to any questions.